1 Laws of Exponents

1. Basic Law of Exponents:

$$b^x * b^y = b^{x+y}$$

2. Difference of Exponents:

$$b^{x-y} = b^x * b^{-y} = \frac{b^x}{b^y}$$

3. Power of a Power:

$$(b^x)^y = b^{(x*y)}$$

4. Power of a Product:

$$(a * b)^x = a^x * b^x$$

5. Power of a Quotient:

$$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$$

6. Definitions:

$$\begin{aligned} b^0 &= 1 \qquad & b^1 = b \\ \sqrt[x]{b} &= b^{\frac{1}{x}} \qquad & b^{-x} = \frac{1}{b^x} \end{aligned}$$

2 Derivatives

1. Basic Derivative Rule:

$$\begin{aligned} \text{If } y(x) &= x^n, \\ \frac{dy}{dx} &= y'(x) = n * x^{(n-1)} \end{aligned}$$

2. Derivatives of a Sum:

$$\begin{aligned} \text{If } y(x) &= f(x) + g(x), \\ \frac{dy}{dx} &= \frac{df(x)}{dx} + \frac{dg(x)}{dx} \quad or \quad y'(x) = f'(x) + g'(x) \end{aligned}$$

3. Product Rule:

$$\begin{aligned} \text{If } y(x) &= f(x) * g(x), \\ \frac{dy}{dx} &= f(x) * \frac{dg(x)}{dx} + g(x) + \frac{df(x)}{dx} \quad or \quad y'(x) = f(x) * g'(x) + g(x) * f'(x) \end{aligned}$$

4. Quotient Rule:

$$\begin{aligned} \text{If } y(x) &= \frac{f(x)}{g(x)}, \\ \frac{dy}{dx} &= \frac{g(x) * \frac{df(x)}{dx} - f(x) * \frac{dg(x)}{dx}}{(g(x))^2} \quad or \quad y'(x) = \frac{g(x) * f'(x) - f(x) * g'(x)}{(g(x))^2} \end{aligned}$$

5. Chain Rule:

$$\begin{aligned} \text{If } y(x) &= f(g(x)), \\ \frac{dy}{dx} &= \frac{df(g(x))}{dx} * \frac{dg(x)}{dx} \quad or \quad y'(x) = f'(g(x)) * g'(x) \end{aligned}$$

3 Equation of a Line

Example: Line Equation

$$2x + y = 100$$

Rearrange terms to get into point-slope formula:

$$y = 100 - 2x$$

where $100 =$ y-intercept and -2 is the slope of the line.

Plotting the line:

1. Find the y-intercept (where $x = 0$, in this case, $y = 100$)
2. Find the x intercept (where $y = 0$, in this case, $x = 50$)
3. Connect the two points

Every line can be written as $AX + BY = C$ where A, B, and C are constants and X and Y are variables.

Note: You should be able to plot this line even if the values of A, B, and C are not specified. Any line can be plotted by simply locating two points on the line. The easiest points to locate are the intercepts.

Example: $ax + by = c$. Find the x- and y- intercept:
If $x = 0$, then $y = \frac{c}{b}$
If $y = 0$, then $x = \frac{c}{a}$
Definition of a slope of a line: (Rise over run):

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

In the reverse, you can derive the equation of a line if you are given two points along it.

Example: Given (x_1, y_1) and (x_2, y_2).

Let (x, y) is any other point on the line. We know the slope of the line, and the slope is constant all along the line. This means we can write:

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Then rearrange:

$$\begin{aligned} y - y_1 &= \frac{y_2 - y_1}{x_2 - x_1} * (x - x_1) \\ y &= \left[y_1 - x_1 * \left(\frac{y_2 - y_1}{x_2 - x_1} \right) \right] + \left(\frac{y_2 - y_1}{x_2 - x_1} \right) * x \end{aligned}$$

Note that the slope is the same form as above, but the term in brackets is the y-intercept:

Example: Given $(1, 2)$, and $(5, 8)$ as points along the line, find the equation for the line. (Answer: $y = \frac{1}{2} + \frac{3}{2}x$).